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Auto-Attitude Stabilization of a Twin-Satellite System Through Very Short Tethers

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The feasibility of achieving passive satellite pitching stabilization through flexible tethers is explored. A twin-satellite platform system comprising two spacecraft halves suitably connected through two pairs of very short tethers is proposed for this purpose. The Lagrangian formulation approach is utilized to develop the governing system of nonlinear ordinary differential equations for the constrained system. These equations are numerically solved along with the equations of constraint. The pitching response characteristics observed establish the strong auto-attitude-stabilization features of the proposed tethered system. The analysis of the first-order attitude perturbation equations for motion-in-the-small around the nominal equilibrium configuration enables prediction of the lower limits of the tether length and offset requirements for stability. These limits may serve as useful guidelines for preliminary system design.

Nomenclature

$\hat{a}$	= $\hat{a}_1$ or $\hat{a}_2$ when $a_1 = a_2$
$\hat{a}_{cr}$	= critical dimensionless tether offsets
a_i	= tether offsets for satellite i, m (Fig. 1)
$\hat{a}_i$	= a_i/L_{ref}
C	= dimensionless tethered satellite system rigidity parameter, $EA/(m_2 L_{ref}\Omega^2)$
EA	= tether modulus of rigidity, N
f	= satellite mass ratio, m_1/m_2
g	= satellite moment of inertia ratio, I_{x_1}/I_{x_2}
I_{k_i}	= principal moments of inertia about k_i axis, $k = x, y, z$, for satellite i, kg-m^2
I_r	= I_{r_1} or I_{r_2} when $I_{r_1} = I_{r_2}$
I_{r_i}	= mass distribution parameter for satellite i, $(I_{y_i} - I_{z_i})/I_{x_i}$
L	= distance between two satellite mass centers, m
L_e	= value of L at equilibrium, i.e., when $\alpha_1 = \alpha_2 = \beta = 0$, m
L_{ij}	= stretched tether lengths, m
L_{ij0}	= unstretched tether lengths, m
L_{ref}	= reference length, $(I_{x_1}/m_2)^{1/2}$, m
L_{t0}	= common unstretched tether lengths L_{ij0}, the offsets being equal, m
L_0	= L when tether strains are zero, $(L_{t0}^2 - a^2)^{1/2}$, m
l	= L/L_{ref}
l_{ij}	= L_{ij}/L_{ref}
l_{ij0}	= L_{ij0}/L_{ref}
l_{t0}	= L_{t0}/L_{ref}
$(l_{t0})_{cr}$	= critical dimensionless tether length
l_0	= L_0/L_{ref}
l_0'	= l' at $\theta = 0$
m_i	= mass of satellite i, kg
R	= orbital radius, m
T	= kinetic energy of the system, N-m
t	= time, s
$U(\epsilon_{ij})$	= 1 for $\epsilon_{ij} \geq 0$ and 0 for $\epsilon_{ij} < 0$
V	= potential energy of the system, N-m
x_i, y_i, z_i	= body coordinate axes for satellite i (Fig. 1)
x_0, y_0, z_0	= coordinate axes in the local vertical frame (Fig. 1)
α_i	= pitch angle for satellite i, deg (Fig. 1)
α_{i0}	= α_i at $\theta = 0$, deg
α_{i0}'	= α_i' at $\theta = 0$, deg
β	= angle denoting relative swing between the two satellite platforms, deg (Fig. 1)
β_0	= β at $\theta = 0$, deg
β_0'	= β' at $\theta = 0$, deg
$\delta(\cdot)$	= perturbed variables $(\cdot)$
ϵ_{ij}	= strain in the jth tether attached to satellite i at its offset point A_{ij} (Fig. 1)
θ	= true anomaly as measured from the reference line (Fig. 1)
λ_{ij}	= Lagrange multipliers
μ	= Earth's gravitational constant, m^3/s^2
Ω	= angular velocity, $(\mu/R^3)^{1/2}$
$(\dot{\cdot})$	= d$(\cdot)$/ dt
$(\cdot)'$	= d$(\cdot)$/dθ
$(\cdot)_i$	= $(\cdot)$ for satellite i, $i = 1, 2$
$(\cdot)_{ij}$	= $(\cdot)$ for jth tether attached to satellite i at its offset point A_{ij}, $i, j = 1, 2$ (Fig. 1)
$\lvert(\cdot)\rvert_{max}$	= maximum amplitudes of $(\cdot)$

Introduction

THE proposal of a Space Shuttle–borne skyhook put forward by Colombo et al.[1] marks the advent of tethered satellite systems (TSS). Satellite attitude stabilization represents one of the several new space applications proposed for the TSS.[2,3] Initially, the efforts focused on achieving passive satellite attitude control through judicious TSS design.[4,5] However, the emphasis later changed to the use of feedback control by regulating tether tension or offsets alone or in combination with established active control devices.[6–9] More recently, Kumar et al.,[10] Kumar,[11] and Kumar and Kumar[12,13] established the feasibility of utilizing a pair of identical tethers to deploy a small pendulum-like auxiliary mass to ensure nearly passive pointing stability for desired arbitrary satellite orientation. The TSS mechanism suggested earlier was found to be quite effective and was successful in bringing down the librational amplitudes even for satellites with adverse mass distribution. However, for stability and adequate attitude response characteristics, this tethered system involved problems of large tether lengths needed (~5 km), as well as its associated weight penalty along with that of the auxiliary mass attached.

To overcome these limitations, it is proposed here to split the spacecraft into halves and suitably connect them through tethers so that each half takes up the role of auxiliary mass for stabilizing the other. Thus, the TSS suggested here (Fig. 1) is composed of the two spacecraft halves connected through very short tethers. The pair of identical tethers originating from the center of mass of each of the two satellites terminates at the other satellite at points located on

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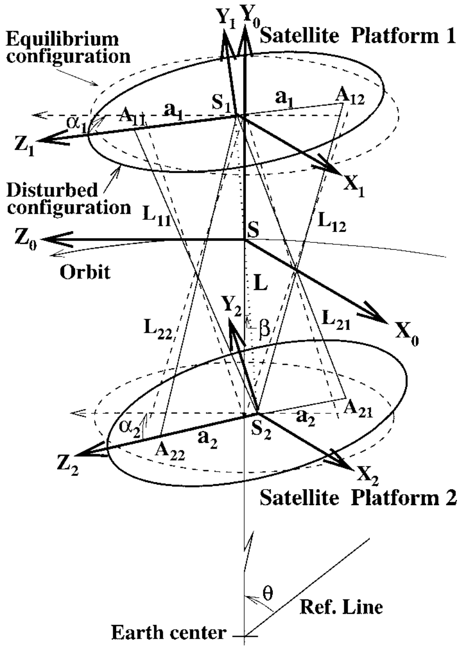

Fig. 1 Geometry of tethered-twin-satellite system.

its principal roll axis symmetrically offset around the mass center. We therefore propose to explore the feasibility of ensuring simultaneous stabilization of the twin-satellite system by judicious TSS design. This study gains added significance in the context of failures of some experimental TSS missions recently undertaken,[14–17] underscoring the need for shorter and stronger tethers for greater mission reliability.

The Lagrangian formulation approach using Lagrange multipliers has been adopted to obtain the governing equations of motion for the constrained system. Subsequently, a first-order pitching perturbation stability analysis is undertaken to determine the necessary design limits on system parameters. Finally, for a detailed assessment of the pitching response characteristics, the set of exact governing equations of motion is numerically integrated.

Formulation

The investigation is initiated by formulating the equations of motion of the proposed TSS moving in a circular orbit. The short extensible tethers connecting the two satellite platforms, made of a light material such as Kevlar®, are assumed to have negligible mass. Consequently, the transverse vibrations are ignored. For exploring the general feasibility of the concept, the case involving in-plane angular motion has been investigated.

The coordinate frame $S{-}x_0y_0z_0$ represents the orbital reference frame. Here, the y_0 axis always points along the local vertical, the x_0 axis is taken along normal to the orbital plane, and the z_0 axis taken along the local horizontal direction completes the right-handed triad. The coordinate axes $S_1{-}x_1y_1z_1$ and $S_2{-}x_2y_2z_2$ are taken along the three centroidal principal axes of satellites 1 and 2, respectively. The angles that the in-plane y_1 and y_2 axes make with the y_0 axis define the two pitching angles α_1 and α_2, respectively. The angle β describes the relative swing between the two platforms. The length L represents the distance joining the two satellite mass centers. The pitch variables α_1, α_2, and β, the length L, and the tether strains ϵ_{11}, ϵ_{12}, ϵ_{21}, and ϵ_{22} constitute the chosen set of generalized coordinates in the present formulation. These variables are, however, subject to the following four constraints:

$$f_{ij} = L_{ij0}(1+\epsilon_{ij}) - \left[a_i^2 + L^2 + (-1)^j 2a_i L \sin(\alpha_i - \beta)\right]^{\frac{1}{2}} = 0 \qquad i, j = 1, 2 \tag{1}$$

To apply the Lagrangian approach for the formulation of the equations of motion, the expressions for the system kinetic energy T as well as the potential energy V are first obtained:

$$T = \frac{1}{2}(m_1+m_2)\dot{\theta}^2R^2 + \frac{1}{2}\left(\frac{m_2}{1+1/f}\right)[\dot{L}^2 + L^2(\dot{\theta}+\dot{\beta})^2] + \frac{1}{2}I_{x_1}(\dot{\theta}+\dot{\alpha}_1)^2 + \frac{1}{2}I_{x_2}(\dot{\theta}+\dot{\alpha}_2)^2 \tag{2}$$

$$V = \sum_{i=1}^{2}\left(-\frac{\mu}{R}m_i + \frac{1}{4}\frac{\mu}{R^3}\left\{\left(I_{x_i}+I_{y_i}+I_{z_i}\right) - 3\left[I_{x_i} + \left(I_{z_i}-I_{y_i}\right)\cos 2\alpha_i\right]\right\}\right) + \frac{1}{2}\left[\frac{\mu}{R^3}\left(\frac{m_2}{1+1/f}\right) \times (1-3\cos^2\beta)L^2 + EA\sum_{i=1}^{2}\sum_{j=1}^{2}L_{ij0}\epsilon_{ij}^2U(\epsilon_{ij})\right] \tag{3}$$

The term $U(\epsilon_{ij})$ in the potential energy expression is simply a unit function, the use of which precludes any negative strain in the tether.[18]

The Lagrangian equations of motion corresponding to the various generalized coordinates indicated earlier may be obtained using the general relation

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial T}{\partial \dot{q}}\right) - \frac{\partial T}{\partial q} + \frac{\partial U}{\partial q} = Q + \sum_{i=1}^{2}\sum_{j=1}^{2}\lambda_{ij}\frac{\partial f_{ij}}{\partial q}, \qquad i, j = 1, 2$$

where q = the generalized coordinate, Q = the generalized force corresponding to the generalized coordinate q, and λ_{ij} = Lagrange multipliers corresponding to the constraints as indicated in Eq. (1).

Substituting the generalized coordinates in the preceding equations and carrying out the algebraic manipulation and nondimensionalization, we get the governing differential equations of motion in the dimensionless form as follows:

$$\alpha_1'' - 3I_{r_1}\sin\alpha_1\cos\alpha_1 + \hat{a}_1[(\lambda_{12}/l_{12}) - (\lambda_{11}/l_{11})]l\cos(\alpha_1-\beta) = 0 \tag{4}$$

$$\alpha_2'' - 3I_{r_2}\sin\alpha_2\cos\alpha_2 + g\hat{a}_2[(\lambda_{22}/l_{22}) - (\lambda_{21}/l_{21})]l\cos(\alpha_2-\beta) = 0 \tag{5}$$

$$\beta'' + 3\sin\beta\cos\beta + 2(1+\beta')(l'/l) - (1+1/f)\{\hat{a}_1[(\lambda_{12}/l_{12}) - (\lambda_{11}/l_{11})]\cos(\alpha_1-\beta) + \hat{a}_2[(\lambda_{22}/l_{22}) - (\lambda_{21}/l_{21})]\cos(\alpha_2-\beta)\}(1/l) = 0 \tag{6}$$

$$l'' - [(1+\beta')^2 - (1-3\cos^2\beta)]l + (1+1/f)[(\lambda_{11}/l_{11}) + (\lambda_{12}/l_{12}) + (\lambda_{21}/l_{21}) + (\lambda_{22}/l_{22})]l + (1+1/f)\{\hat{a}_1[(\lambda_{12}/l_{12}) - (\lambda_{11}/l_{11})]\sin(\alpha_1-\beta) + \hat{a}_2[(\lambda_{22}/l_{22}) - (\lambda_{21}/l_{21})]\sin(\alpha_2-\beta)\} = 0 \tag{7}$$

$$\lambda_{ij} = \left(\frac{EA}{m_2L_{\mathrm{ref}}\Omega^2}\right)\epsilon_{ij}U(\epsilon_{ij}), \qquad i, j = 1, 2 \tag{8}$$

Note that, the constraints being linear in tether strains ϵ_{ij}, it has been possible to obtain explicit expressions for the Lagrange multipliers. This fact leads to considerable simplification in the process of numerical integration, as elaborated later.

The preceding set of nonlinear and coupled equations is quite complex and does not seem to be amenable to analytical treatment for solution. It is therefore proposed to undertake the first-order perturbation stability analysis to develop design guidelines for system stability.

Stability Analysis

For stability analysis, we obtain from the governing equations of motion the first-order perturbation equations for motion-in-the-small around the nominal system equilibrium configuration. For convenience of practical implementation, we also take the tether offsets a_1 and a_2 to be equal so that

$$\hat{a}_1 = \hat{a}_2 = \hat{a}$$

In view of the finite tensions always present in the tethers in the nominal equilibrium configuration and also during the motion-in-the-small, the unit function

$$U(\epsilon_{ij}) = 1, \qquad i, j = 1, 2$$

Using the preceding substitutions and neglecting second- and higher-order terms in the perturbed variables, we get the following linearized equations of motion:

$$\delta\alpha_1'' + C_1\delta\alpha_1 + C_2\delta\beta = 0, \qquad \delta\alpha_2'' + C_3\delta\alpha_2 + C_4\delta\beta = 0$$
$$\delta\beta'' + C_5\delta\alpha_1 + C_6\delta\alpha_2 + C_7\delta\beta + C_8\delta l' = 0 \tag{9}$$
$$\delta l'' + C_9\delta l + C_{10}\delta\beta' = 0$$

where

$$C_1 = -3I_{r_1} - C_2, \qquad C_2 = -2C\hat{a}^2/l_{t0}, \qquad C_3 = -3I_{r_2} - gC_2$$
$$C_4 = gC_2, \qquad C_5 = C_6 = -2(1 + 1/f)C\hat{a}^2/l_{t0}^3$$
$$C_7 = 3 + (1 + 1/f)4C\hat{a}^2/l_{t0}^3, \qquad C_8 = 2/l_{t0}$$
$$C_9 = -3 + 4(1 + 1/f)C/l_{t0}, \qquad C_{10} = -2l_{t0}$$

The characteristic equation for the system now takes the form

$$\left(S^2 + C_1\right)\left(S^2 + C_3\right)\left[\left(S^2 + C_7\right)\left(S^2 + C_9\right) - S^2C_8C_{10}\right]$$
$$- C_4C_6\left(S^2 + C_1\right)\left(S^2 + C_9\right) - C_2C_5\left(S^2 + C_3\right)\left(S^2 + C_9\right) = 0 \tag{10}$$

For a conservative analysis, we consider the situation of the inherently most unstable mass distributions for the two satellites around their respective mass centers. For such systems, we therefore take

$$I_{r_1} = I_{r_2} = 1$$

Besides, an order-of-magnitude analysis of the various coefficients in the characteristic equation shows that the term $C_8C_{10}S^2$ present before can be ignored without significantly affecting the eigenvalues of the system. With a view to facilitating development of stability conditions in neat analytical form, we sacrifice rigor by treating only the particular case when $g = 1$. Consequently, the system characteristic equation simplifies to

$$\left(S^2 + C_1\right)\left(S^2 + C_9\right)\left[S^4 + (C_1 + C_7)S^2 + (C_1C_7 - 2C_2C_5)\right] = 0 \tag{11}$$

The conditions for system stability can now be stated as

$$C_1, C_9 > 0, \qquad (C_1 + C_7) > 0, \qquad (C_1C_7 - 2C_2C_5) > 0 \tag{12}$$

On substituting the expressions for the various parameters in the preceding inequalities, the approximate conditions for stability can be summarized as follows:

$$(2C\hat{a}^2/3) > l_{t0} > [2(1 + 1/f)]^{\frac{1}{2}}$$
$$C > 3l_{t0}/[4(1 + 1/f)] \tag{13}$$
$$\hat{a} > [3l_{t0}/(2C)]^{\frac{1}{2}}\left[1 - 2(1 + 1/f)/l_{t0}^2\right]^{-\frac{1}{2}}$$

Results and Discussion

With a view to assessing the effectiveness for stabilization of the two satellites through the proposed tether attachments, the detailed system response is numerically simulated using Eqs. (1) and (4–8). For known starting values of variables α_1, α_2, β, and l at each step of numerical integration, we first solve for tether strains ϵ_{ij} using the constraint relations (1) rewritten in explicit form as follows:

$$\epsilon_{ij} = \left[\hat{a}_i^2 + l^2 + (-1)^j 2\hat{a}_i l \sin(\alpha_i - \beta)\right]^{\frac{1}{2}} / l_{ij0} - 1, \qquad i, j = 1, 2 \tag{14}$$

The substitution of these tether strains in Eqs. (8) enables determination of the four Lagrange multipliers λ_{ij}. The values of the tether strains and the Lagrange multipliers thus explicitly available are utilized to compute the new values of the variables at the end of the step through numerical integration of the set of the differential equations (4–7). The integration is based on NAG routine D02CBF using the variable-step Adams method.

Two different classes of system models are considered for the simulation: In class 1, the system is composed of two identical satellites connected by tethers, and hence

$$f = (m_1/m_2) = 1, \qquad g = \left(I_{x_1}/I_{x_2}\right) = 1, \qquad I_{r_1} = I_{r_2} = I_r$$

There is no restriction on the size of the satellites or their mass distributions, however. In class 2, the system satellites are not identical, i.e., f and/or g may not necessarily be equal to 1. Similarly, mass distribution parameters for the two satellites may or may not be the same. For illustration, a particular case has been considered with the parameters values as follows:

$$f = 2, \qquad g = 5$$

Several typical combinations of dimensionless parameters are considered for the presentation of attitude response simulation results. Of the various parameters influencing the system attitude behavior, the parameter C may be of special significance. The values of this parameter are dependent on the tether rigidity modulus EA, the first satellite platform moment of inertia I_{x_1}, the second satellite platform mass m_2, and the orbital angular velocity Ω. For a near-Earth satellite, its practical values, assuming thin tethers and feasible twin-satellite systems, may range from, e.g., 10^3 to 10^9. For the geosynchronous orbits, the corresponding parameter values would be higher, e.g., in the range from 10^5 to 10^{11}. Evidently, the larger values signify systems with greater overall relative rigidity. That is why the parameter C has been referred to as the TSS rigidity parameter in the present investigation.

In the particular case of systems employing massless rigid rods as tethers, the resulting librational behavior has been thoroughly investigated in the past. Here, the assumed initial pitch angle and pitch rate disturbances in the three librational degrees of freedom α_1, α_2, and β would necessarily be identical, and so would be the subsequent attitude response. For the tether lengths above the minimum predicted from the rigid system stability criterion, the librational response to even significant initial pitch momentum disturbances was found to have rather low amplitudes. The system attitude behavior was found to improve further with an increase in the tether length chosen.

In the present investigation, attention is focused on flexible systems wherein the three initial pitch angles and/or their rates, as well as their values at any subsequent point of time, can be, in general, quite different. For simulating the response, the initial values of pitching angles α_{10}, α_{20}, and β_0 are taken to be zero, whereas their corresponding initial rates α_{10}', α_{20}', and β_0' are assumed to have arbitrary values.

Figure 2 shows the effect of the nature of initial pitching rates on the librational response for class 1 and 2 satellite systems. Interestingly, regardless of the nature of pitching rates, the amplitude of motion remains remarkably low in all cases. This may be attributed to the generation of control moments through differential changes in tether tensions as a result of satellite attitude drifts. In general, the presence of pitch rate disturbances β_0' leads to significantly larger librational amplitudes. In situations with three initial pitch rates differing significantly, the two satellite librational pitch characteristics may be quite different.

Figure 3 presents the response plots showing the effect of the two satellite mass distribution parameters I_{r_1} and I_{r_2} on satellite librational response. These plots clearly establish the importance of the proposed tether attachments, which effectively limit the system

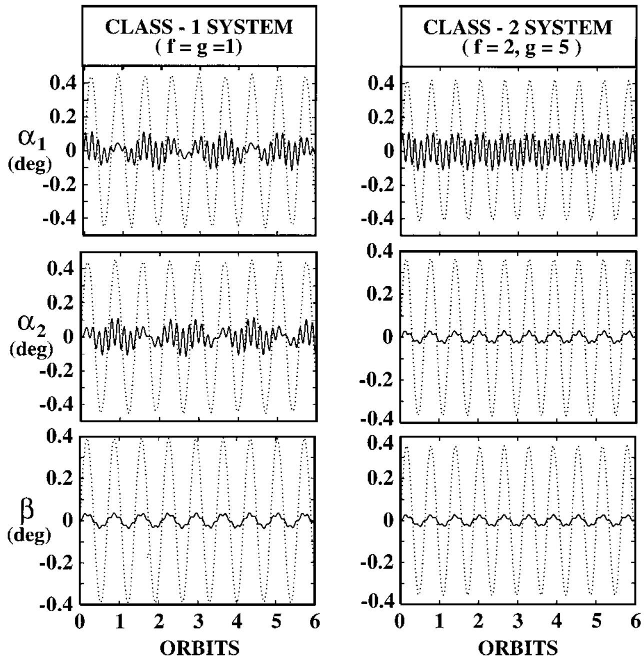

Fig. 2 Response plots showing influence of nature of pitching rates on pitch response; $C = 10^4$, $I_r = 1$, $l_{t0} = 5$, $\hat{a} = 0.1$, and $\alpha_{i0} = \beta_0 = 0$: ——, $\alpha'_{10} = 0.01$, $\alpha'_{20} = \beta'_0 = l'_0 = 0$; and ·····, $\alpha'_{10} = \alpha'_{20} = \beta'_0 = 0.01$, $l'_0 = 0$.

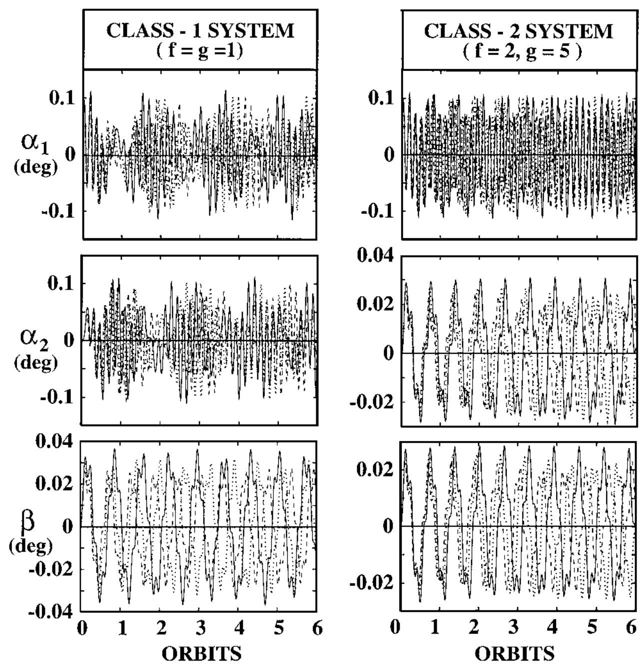

Fig. 3 Typical system pitch response as affected by the two satellite mass distributions; $C = 10^4$, $l_{t0} = 5$, $\hat{a} = 0.1$, $\alpha_{i0} = \beta_0 = 0$, $\alpha'_{10} = 0.01$, and $\alpha'_{20} = \beta'_0 = l'_0 = 0$: ——, $I_r = 1$; ·····, $I_r = 0$; and – – –, $I_r = -1$.

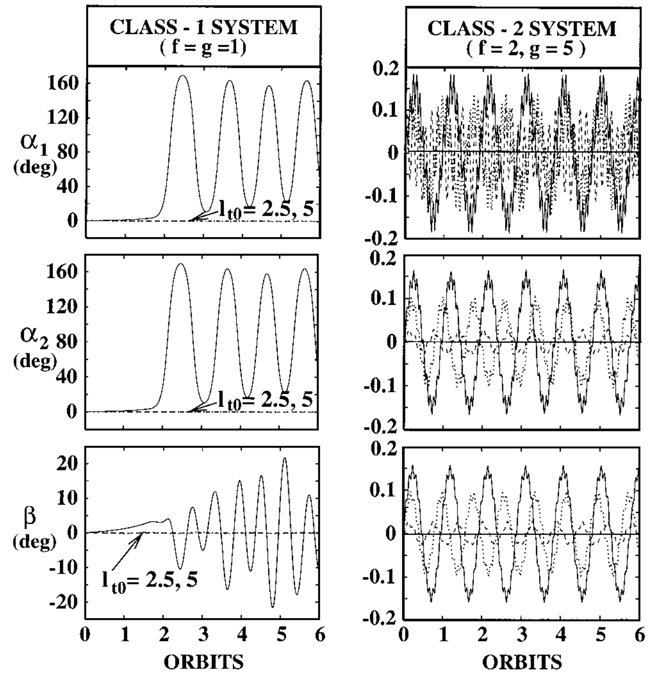

Fig. 4 Typical system pitch response as influenced by tether lengths; $C = 10^4$, $I_r = 1$, $\hat{a} = 0.1$, $\alpha_{i0} = \beta_0 = 0$, $\alpha'_{10} = 0.01$, and $\alpha'_{20} = \beta'_0 = l'_0 = 0$: ——, $l_{t0} = 2$; ·····, $l_{t0} = 2.5$; and – – –, $l_{t0} = 5$.

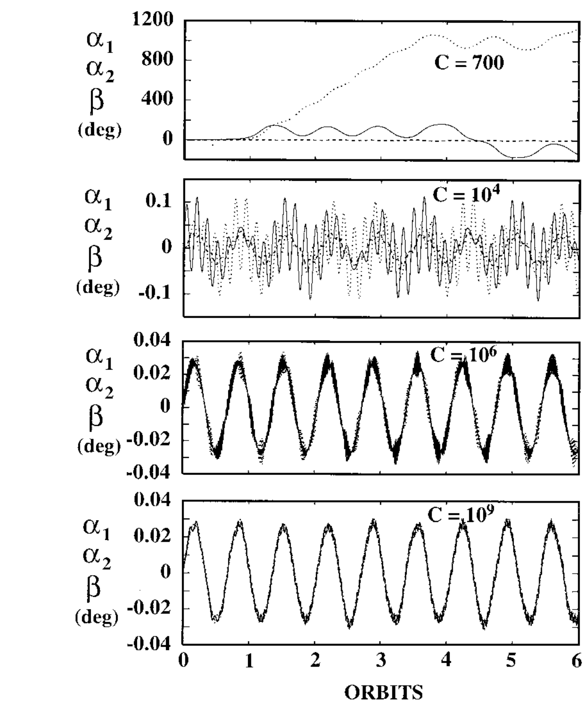

Fig. 5 Typical system pitch response as affected by the TSS rigidity parameter C; $I_r = 1$, $l_{t0} = 5$, $\hat{a} = 0.1$, $\alpha_{i0} = \beta_0 = 0$, $\alpha'_{10} = 0.01$, and $\alpha'_{20} = \beta'_0 = l'_0 = 0$: ——, α_1; ·····, α_2; and – – –, β.

librational amplitudes to much lower values. Even for the inherently most unstable satellite configurations, librational amplitudes are practically the same as those for other stable or unstable combinations. The effect of the most adverse satellite mass distribution is one of only slightly enhanced oscillation amplitudes. This result was not unexpected, however, as the stabilizing differential tether tension moment simply overwhelms the weak gravity gradient moments.

The choice of dimensionless tether length parameter is rather important and can have a significant effect on response (Fig. 4). The case $l_{t0} = 2$ is of particular interest as, according to the stability criteria here, it represents the case of instability for the system of class 1 and the case of stable librations for that of class 2. The response for class 2 is found to be stable, whereas that for class 1 shows instability as expected in the presence of finite initial disturbances. Taking l_{t0} to be higher, e.g., 2.5, which is well above the critical length for class 1 and 2 systems, ensures librational stability in all cases considered. A further increase in length leads to only a marginal improvement in satellite attitude behavior with reduced oscillation amplitudes.

It is now proposed to examine the effect of the choice of the TSS rigidity parameter value on system dynamics. The detailed simulation shows that, for $C = 700$, a value below its minimum/critical value of 750 as predicted by stability analysis, the system does indeed show unstable behavior (Fig. 5). For the more realistic and feasible higher value $C = 10^4$, the system becomes stable with fairly

Table 1 Minimum tether length requirements for various twin-satellite systems ($C = 10^4$ and $\hat{a} = 0.1$)

	Class 1			Class 2	
System data	Light-weight satellites	Medium-weight satellites	Heavy-weight satellites	Lightweight–medium-weight combinations	Shuttle-satellite system
m_1, kg	500	1000	149.8×10^3	1000	149.8×10^3
m_2, kg	500	1000	149.8×10^3	500	74.9×10^3
I_{x_1}, kg m^2	100	500	1.32×10^7	500	1.32×10^7
I_{x_2}, kg m^2	100	500	1.32×10^7	100	2.64×10^6
I_{y_1}, kg m^2	315	1000	9.98×10^7	315	9.98×10^7
I_{y_2}, kg m^2	315	1000	9.98×10^7	100	6.63×10^6
I_{z_1}, kg m^2	215	500	9.60×10^7	500	9.60×10^7
I_{z_2}, kg m^2	215	500	9.60×10^7	215	5.56×10^6
I_{r_1}	1.0	1.0	0.3	1.0	0.3
I_{r_2}	1.0	1.0	0.3	1.0	0.3
L_{ref}, m	0.45	0.71	9.39	1.00	13.28
$(L_0)_{\min}$, m	0.9	1.5	19.0	2.0	23.0

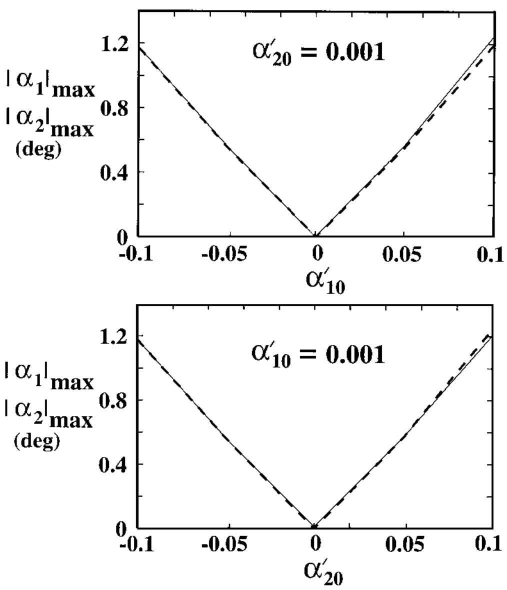

Fig. 6 System plots showing the influence of the nature of initial pitching rates on librational response for class 1 satellite systems ($f = g = 1$); $C = 10^4$, $I_r = 1$, $l_{t0} = 5$, $\hat{a} = 0.1$, and $\alpha_{i0} = \beta_0 = \beta'_0 = l'_0 = 0$: ——, $|\alpha_1|_{\max}$, and – – –, $|\alpha_2|_{\max}$.

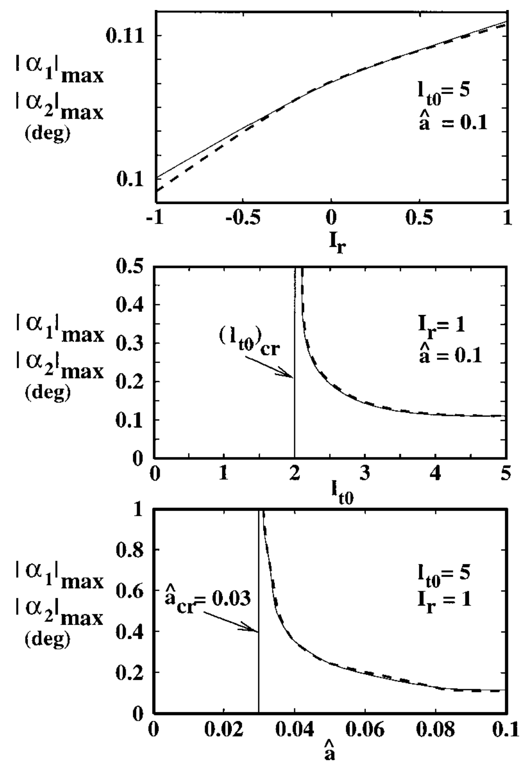

Fig. 7 System plots showing the influence of various design parameters on librational response for class 1 satellite systems ($g = f = 1$); $C = 10^4$, $\alpha_{i0} = \beta_0 = 0$, $\alpha'_{10} = 0.01$, and $\alpha'_{20} = \beta'_0 = l'_0 = 0$: ——, $|\alpha_1|_{\max}$, and – – –, $|\alpha_2|_{\max}$.

modest amplitudes of pitching oscillations. In general, the three librational angles α_1, α_2, and β appear to maintain their separate entity with large differences in phase angles as well as in amplitudes. Thus, in effect, the attitude response appears to correspond to a flexible system. In general, an increase in the value of C leads to a significant reduction in librational amplitudes. Its other important consequence is the one of the three librational responses approaching each other with time. The larger the value of C, the closer and faster they get to each other. These trends are particularly significant in initial stages in cases when the initial pitching disturbances in the three librational degrees of freedom are widely different. Thus, for very high values of C, the three responses appear to merge throughout, implying virtually a rigid system behavior. When the three initial pitching rates are equal, the consequent pitching responses may appear to be practically identical even for low values of C.

The preceding results bring out the powerful auto-attitude-stabilization features of twin-satellite systems. Although these features are equally valid for classes 1 and 2, we focus our attention on the particular case of a class 1 system with flexible tethers for in-depth attitude response studies. The vast amount of response information generated over a wide range of design parameters is summarized in the form of system plots (Figs. 6 and 7). The importance of these plots cannot be overemphasized as these facilitate a judicious choice of the various system parameters at a preliminary design stage. Figure 6 shows the effect of varying initial pitching rates on maximum satellite librational amplitudes. Note that, even for relatively large initial disturbances of, e.g., 0.05 in the satellite pitch rates, the amplitudes of motion are merely fractions of a degree. For disturbances of much smaller magnitudes, normally expected in practical situations, the satellite attitude drifts may be much lower and well within acceptable limits.

Figure 7 presents the effect of varying mass distribution parameters, tether lengths, and tether offsets. The oscillation amplitudes remain fairly low throughout regardless of the mass distributions. A slight increase in amplitudes is observed as the individual platform mass distributions change from the most stable to most unstable gravity gradient configurations. In effect, the tether attachments enable the system to practically do away with mass distribution constraints altogether. In contrast, the dimensionless tether lengths have a rather significant effect on response, and hence these must be judiciously chosen so as to ensure desirable system attitude characteristics. A choice of l_{t0} slightly greater than the critical length $(l_{t0})_{\text{cr}} = 2$, as predicted from Eq. (13), is normally adequate for acceptable librational response. Increasing lengths further results in only a marginal improvement in system attitude performance.

Figure 7 also shows the influence of dimensionless tether offsets on satellite attitude performance. It is only when the tether offsets chosen are very low that the librational amplitudes may be relatively large and unacceptable. Below the critical tether offset, $\hat{a}_{cr} = 0.03$, as predicted by the stability analysis for the system under consideration, the system indeed becomes unstable. The tether offsets well above the critical value, however, have little effect on librational amplitudes. For the system under consideration, the offsets on the order of a fraction of a meter may be adequate for the TSS design.

The information concerning the minimum tether length needed for pointing stability of various satellite combinations is presented in Table 1, which covers practically all possible bisatellite platform configurations composed of a light-, medium-, or heavyweight satellite tethered to a similar satellite or to the one with very different mass and inertia properties. In general, the tether lengths required increase with increase in size of the two satellites. Very short tethers, no longer than 20–25 m, are required even when both the satellites are large and heavy. That makes the proposed concept particularly attractive.

Concluding Remarks

This paper presents the concept of self-induced attitude stabilization for the twin-satellite system using short and flexible tethers. The results of the analysis and subsequent attitude response simulation of the exact governing equations of motion over a wide range of design parameters clearly show that judicious tether attachments between the two satellite platforms not only ensure system attitude stability but also bring down the librational amplitudes to rather low values even in the presence of large initial pitch rate disturbances. An increase in the TSS rigidity parameter leads to further improvement in system attitude response.

The importance of the first-order perturbation stability analysis undertaken here cannot be overemphasized, as this leads to useful guidelines for system design. The system plots presented can further aid in the judicious choice of system parameters. Of course, the final design values chosen should be duly validated through detailed numerical simulation of the governing nonlinear equations of motion.

It appears quite conceivable to visualize distribution of various subsystems of a single satellite into two subsatellites slightly separated from each other and kept in their position through very short tethers. Such a system would effectively form a single satellite at launch. The usual TSS deployment procedures could be utilized to achieve separation of the two satellites. Alternatively, the weak springs initially provided to hold the two satellites together may be released to achieve the same objective.

Finally, it is hoped that the present investigation may serve as a catalyst for more detailed studies, pave the way for designers to solve the associated problems of implementation, and thus ensure realization of the proposed concept.

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